

Table 1 Lift and moment coefficients for pitching and plunging of a typical jet transport wing at a Mach number of 0.8

| k | Pitching | | Plunging | |
|-------|----------------------|----------------------|------------------------------|------------------------------|
| | \bar{C}_z/α_0 | \bar{C}_m/α_0 | $\bar{C}_z/ikh_0(2/\bar{c})$ | $\bar{C}_m/ikh_0(2/\bar{c})$ |
| 0.001 | $-5.856 + i0.0067$ | $-0.5653 - i0.0023$ | $-5.856 + i0.0126$ | $-0.5653 + i0.0009$ |
| 0.002 | $-5.856 + i0.0134$ | $-0.5652 - i0.0046$ | $-5.856 + i0.0252$ | $-0.5652 + i0.0018$ |
| 0.004 | $-5.855 + i0.0267$ | $-0.5651 - i0.0093$ | $-5.855 + i0.0505$ | $-0.5651 + i0.0035$ |
| 0.005 | $-5.855 + i0.0333$ | $-0.5650 - i0.0116$ | $-5.854 + i0.0630$ | $-0.5650 + i0.0044$ |
| 0.010 | $-5.849 + i0.0660$ | $-0.5643 - i0.0233$ | $-5.847 + i0.1254$ | $-0.5642 + i0.0087$ |
| 0.020 | $-5.826 + i0.1278$ | $-0.5615 + i0.0473$ | $-5.820 + i0.2459$ | $-0.5612 + i0.0168$ |
| 0.040 | $-5.749 + i0.2288$ | $-0.5524 - i0.0981$ | $-5.724 + i0.4609$ | $-0.5514 + i0.0297$ |
| 0.050 | $-5.698 + i0.2662$ | $-0.5466 - i0.1249$ | $-5.662 + i0.5529$ | $-0.5452 + i0.0346$ |
| 0.100 | $-5.398 + i0.3251$ | $-0.5123 - i0.2730$ | $-5.278 + i0.8558$ | $-0.5089 + i0.0433$ |

Table 2 Estimates of C_{z_α} , C_{m_α} , $C_{z_{\dot{\alpha}}}$, $C_{m_{\dot{\alpha}}}$, C_{z_q} and C_{m_q} based on a single value of k

| k | C_{z_α} | C_{m_α} | $C_{z_{\dot{\alpha}}}$ | $C_{m_{\dot{\alpha}}}$ | C_{z_q} | C_{m_q} |
|-------|----------------|----------------|------------------------|------------------------|-----------|-----------|
| 0.0 | -5.856 | -0.5653 | - | - | -5.947 | -3.206 |
| 0.001 | -5.856 | -0.5653 | 12.63 | 0.8883 | -5.947 | -3.206 |
| 0.002 | -5.856 | -0.5652 | 12.63 | 0.8878 | -5.946 | -3.206 |
| 0.004 | -5.855 | -0.5651 | 12.62 | 0.8861 | -5.945 | -3.206 |
| 0.005 | -5.855 | -0.5650 | 12.61 | 0.8848 | -5.944 | -3.206 |
| 0.010 | -5.849 | -0.5643 | 12.54 | 0.8744 | -5.936 | -3.205 |
| 0.020 | -5.826 | -0.5615 | 12.30 | 0.8388 | -5.907 | -3.203 |
| 0.040 | -5.749 | -0.5524 | 11.52 | 0.7432 | -5.803 | -3.195 |
| 0.050 | -5.698 | -0.5466 | 11.06 | 0.6916 | -5.734 | -3.190 |
| 0.100 | -5.398 | -0.5123 | 8.56 | 0.4326 | -5.307 | -3.162 |

$$C_{m_{q1,3}} = (I/\bar{c}^3) [x_l - x] [C_{h_s}] \{x_3 - x\}^2 \quad (17)$$

The moment about the first moment axis for pitching about the second pitch axis is

$$\begin{aligned} C_{m_{q1,4}} &= (I/\bar{c}^3) [x_l - x] [C_{h_s}] \{x_4 - x\}^2 \\ &= C_{m_{q1,3}} + 2C_{m_{\alpha_1}} (\Delta x_3/\bar{c}) \end{aligned} \quad (18)$$

The moment about the second moment axis for pitching about the first pitch axis is obviously

$$C_{m_{q2,3}} = C_{m_{q1,3}} + C_{z_{q3}} (\Delta x_1/\bar{c}) \quad (19)$$

Finally, the moment about the second moment axis for pitching about the second pitch axis is

$$\begin{aligned} C_{m_{q2,4}} &= (I/\bar{c}^3) [x_2 - x] [C_{h_s}] \{x_4 - x\}^2 \\ &= C_{m_{q2,3}} + 2C_{m_{\alpha_2}} (\Delta x_3/\bar{c}) \end{aligned} \quad (20)$$

Equations (14), (16), (18), and (20) are variously found in the USAF DATCOM.⁷

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References

- ¹Rodden, W.P. and Giesing, J.P., "Application of Oscillatory Aerodynamic Theory to Estimation of Dynamic Stability Derivatives," *Journal of Aircraft*, Vol. 7, May-June 1970, pp. 272-275.
- ²Giesing, J.P., Kalman, T.P., and Rodden, W.P., "Subsonic Unsteady Aerodynamics for General Configurations; Part II—Application of the Doublet-Lattice Method and the Method of Images to Lifting-Surface/Body Interference," AFFDL-TR-71-5, Part II, April 1972.

³Rodden, W.P., Harder, R.L., and Bellinger, E.D., "Aeroelastic Addition to NASTRAN," NASA CR 3094, March 1979.

⁴Stahl, B., Kalman, T.P., Giesing, J.P., and Rodden, W.P., "Aerodynamic Influence Coefficients for Oscillatory Planar Lifting Surfaces by the Doublet Lattice Method for Subsonic Flows Including Quasi-Steady Fuselage Interference," McDonnell Douglas Corp., Rept. DAC-67201, Oct. 1968.

⁵Giesing, J.P., Kalman, T.P., and Rodden, W.P., "Subsonic Unsteady Aerodynamics for General Configurations; Part I—Direct Application of the Nonplanar Doublet Lattice Method," AFFDL-TR-71-5, Part I, Nov. 1971.

⁶Rodden, W.P. and Revell, J.D., "Status of Unsteady Aerodynamic Influence Coefficients," Paper FF-33, Institute of the Aeronautical Sciences, 1962; preprinted as Rept. TDR-930-(2230-09)TN-2, Aerospace Corp., 1961.

⁷Finck, R.D. and Hoak, D.E., "USAF Stability and Control DATCOM," Air Force Flight Dynamics Laboratory, Flight Control Division, revised 1976.

Aeroelastic Divergence of Unrestrained Vehicles

William P. Rodden*

La Cañada Flintridge, California

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REFERENCE 1 presented an analysis of an unrestrained vehicle that is in flight and in quasi-static equilibrium by virtue of its inertial forces. A singularity appeared in the formulation that was identified as a physical aeroelastic divergence. This is not the case. Rather than a divergence, it is only a singularity that appears at the dynamic pressure at which the principal axis (i.e., the mean longitudinal reference

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*Consulting Engineer. Associate Fellow AIAA.

axis of Milne² when transverse displacement is the primary degree of freedom) remains aligned with the freestream as the vehicle deforms. This becomes apparent by following the alternative derivation of Letsinger³ to obtain the unrestrained longitudinal stability derivatives referenced to the principal axis.

In the following, the subscript *s* denotes values at the origin of the structural support system to which the structural influence coefficients (SIC's) are referred. The structural support system defines the attached axes of Milne² as tangent, normal, and binormal to the fuselage deflection curve. Consider the symmetrical trim problem of a flexible vehicle. We first consider the restrained vehicle for which the trimmed lift and moment coefficients, referred to some reference area *S* and chord length \bar{c} , are given by

$$\begin{Bmatrix} C_z \\ C_m \end{Bmatrix} = \begin{bmatrix} C_{z\alpha} & C_{z\delta} \\ C_{m\alpha} & C_{m\delta} \end{bmatrix} \begin{Bmatrix} \alpha_s \\ \delta \end{Bmatrix} + \begin{bmatrix} C_{z\ddot{z}} & C_{z\ddot{\theta}} \\ C_{m\ddot{z}} & C_{m\ddot{\theta}} \end{bmatrix} \begin{Bmatrix} \ddot{z}_s/g \\ \ddot{\theta}_s \bar{c}/2g \end{Bmatrix} \quad (1)$$

in which α_s and δ are the trim angle of attack and control surface incidence, \ddot{z}_s and $\ddot{\theta}_s$ are the plunging and pitching accelerations of the principal axis at the longitudinal location of the origin of the support system, and *g* is the acceleration of gravity.

If we consider the accelerations to be prescribed by the equations of motion, rather than as arbitrary quantities, we have

$$\begin{bmatrix} M & S_s \\ S_s & I_s \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{\theta}_s \end{Bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \bar{c} \end{bmatrix} \begin{Bmatrix} C_z \\ C_m \end{Bmatrix} qS \quad (2)$$

Eliminating the accelerations between Eqs. (1) and (2) leads to

$$\begin{Bmatrix} C_z \\ C_m \end{Bmatrix} = \left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - qS \begin{bmatrix} C_{z\ddot{z}} & C_{z\ddot{\theta}} \\ C_{m\ddot{z}} & C_{m\ddot{\theta}} \end{bmatrix} \begin{bmatrix} I/g & 0 \\ 0 & \bar{c}/2g \end{bmatrix} \begin{bmatrix} M & S_s \\ S_s & I_s \end{bmatrix} \right)^{-1} \times \begin{bmatrix} I & 0 \\ 0 & \bar{c} \end{bmatrix} \begin{bmatrix} C_{z\alpha} & C_{z\delta} \\ C_{m\alpha} & C_{m\delta} \end{bmatrix} \begin{Bmatrix} \alpha_s \\ \delta \end{Bmatrix} \quad (3a)$$

$$\equiv \begin{bmatrix} C'_{z\alpha} & C'_{z\delta} \\ C'_{m\alpha} & C'_{m\delta} \end{bmatrix} \begin{Bmatrix} \alpha_s \\ \delta \end{Bmatrix} \quad (3b)$$

The new primed stability derivatives of Eq. (3b) may be regarded as unrestrained stability derivatives, but are still dependent on the selection of the support system for the SIC's.

A set of stability derivatives that is invariant with the SIC support system is found by measuring the angle of attack with respect to the principal axis rather than the structural axis, as shown in Fig. 1. Deflection calculations permit writing

$$\alpha_p = \alpha_s + \beta \quad (4)$$

where

$$\beta = \left(\frac{\partial \beta}{\partial \alpha} \right) \alpha_s + \left(\frac{\partial \beta}{\partial \delta} \right) \delta \quad (5)$$

and the partial derivative coefficients are found from the rotations of the principal axis resulting from the loadings for unit values of α_s and δ . We combine Eqs. (4) and (5) to read

$$\begin{Bmatrix} \alpha_p \\ \delta \end{Bmatrix} = \begin{bmatrix} I + \frac{\partial \beta}{\partial \alpha} & \frac{\partial \beta}{\partial \delta} \\ 0 & I \end{bmatrix} \begin{Bmatrix} \alpha_s \\ \delta \end{Bmatrix} \quad (6)$$

or

$$\begin{Bmatrix} \alpha_s \\ \delta \end{Bmatrix} = \frac{I}{I + \frac{\partial \beta}{\partial \alpha}} \begin{bmatrix} I & -\frac{\partial \beta}{\partial \delta} \\ 0 & I + \frac{\partial \beta}{\partial \alpha} \end{bmatrix} \begin{Bmatrix} \alpha_p \\ \delta \end{Bmatrix} \quad (7)$$

Substituting Eq. (7) into Eq. (3b) leads to

$$\begin{Bmatrix} C_z \\ C_m \end{Bmatrix} = \frac{I}{I + \frac{\partial \beta}{\partial \alpha}} \begin{bmatrix} C'_{z\alpha} & C'_{z\delta} \\ C'_{m\alpha} & C'_{m\delta} \end{bmatrix} \times \begin{bmatrix} I & -\frac{\partial \beta}{\partial \delta} \\ 0 & I + \frac{\partial \beta}{\partial \alpha} \end{bmatrix} \begin{Bmatrix} \alpha_p \\ \delta \end{Bmatrix} \quad (8a)$$

$$\equiv \begin{bmatrix} C''_{z\alpha} & C''_{z\delta} \\ C''_{m\alpha} & C''_{m\delta} \end{bmatrix} \begin{Bmatrix} \alpha_p \\ \delta \end{Bmatrix} \quad (8b)$$

The new doubly primed stability derivatives become the unrestrained stability derivatives referred to the principal axis and are invariant with the choice of SIC support system. These are the stability derivatives calculated by FLEXSTAB⁴.

In going from Eq. (3b) to Eq. (8b) the critical parameter is the derivative $\partial \beta / \partial \alpha$. Its value depends on the dynamic pressure and, depending on the configuration, can increase or decrease with dynamic pressure (its value at zero dynamic pressure is zero). If it reaches a value of -1.0 , the stability derivatives become infinite and reverse their signs at higher dynamic pressures. The variation $\partial \beta / \partial \alpha$ is shown in Table 1 using the example of Ref. 1 with $\lambda = 30$ deg and $r = 0.25$. The value of $qc^4 a_0 / GJ$, for which $\partial \beta / \partial \alpha = -1.0$, is 0.992; $\partial \beta / \partial \alpha$ also becomes singular where Eq. (3) becomes infinite (at $qc^4 a_0 / GJ = 1.012$ in this example) but this has no practical significance, just as the restrained divergence dynamic pressure ($qc^4 a_0 / GJ = 0.887$ in the example) has no physical meaning for the unrestrained vehicle other than to suggest potential dynamic stability problems.

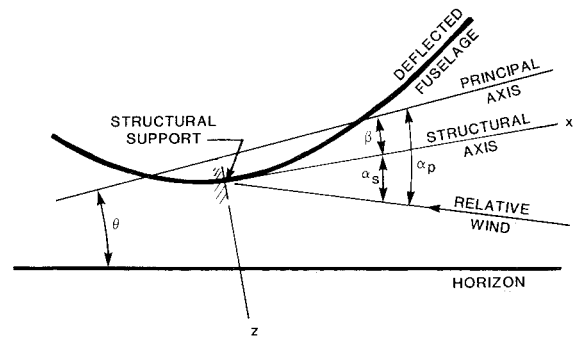


Fig. 1 Geometry of deformed flight vehicle.

Table 1 Variation of $\partial \beta / \partial \alpha$ with dynamic pressure

| $qc^4 a_0 / GJ$ | $\partial \beta / \partial \alpha$ |
|-----------------|------------------------------------|
| 0.0 | 0.0 |
| 0.25 | -0.007 |
| 0.50 | -0.020 |
| 0.75 | -0.058 |
| 0.90 | -0.162 |
| 0.95 | -0.310 |
| 0.98 | -0.622 |
| 0.99 | -0.918 |
| 0.992 | -1.000 |
| 1.00 | -1.715 |
| 1.012 | $\pm \infty$ |
| 1.02 | +2.501 |

The identity of the dynamic pressures, for which both $\partial\beta/\partial\alpha = -1.0$ and Eq. (18) of Ref. 1 is singular, has not been demonstrated analytically but only numerically. However, numerous numerical experiments have shown that this is always the case, and we conclude that the "divergence" in Ref. 1 is not physical but is only a mathematical property of the principal axis. The limited study of Ref. 5 now suggests that divergence of an unrestrained vehicle should always be investigated by methods of dynamic stability analysis.

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References

- ¹Rodden, W.P., "Aeroelastic Divergence of Unrestrained Vehicles," *Journal of Aircraft*, Vol. 18, Dec. 1981, pp. 1072-1073.
- ²Milne, R.D., "Dynamics of the Deformable Aeroplane," British A.R.C. R&M 3345, 1964.
- ³Letsinger, G.R., "Stability and Control Derivatives Including Aeroelastic Effects," unpublished Document No. D6A 11589-1TN, The Boeing Aerospace Co., Feb. 1969.
- ⁴Dusto, A.R., et al. "A Method for Predicting the Stability Derivatives of an Elastic Airplane; Vol. I - FLEXSTAB Theoretical Description," NASA CR-114712, Oct. 1974 or AFFDL TR-74-91, Vol. I, Nov. 1974.
- ⁵Rodden, W.P., and Bellinger, E.D., "Unrestrained Aeroelastic Divergence in a Dynamic Stability Analysis," *Journal of Aircraft*, Vol. 19, Sept. 1982, pp. 796-797.

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